Phenomena Connected with Turbulence in the Lower Atmosphere.

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The object of the present paper is to bring together some of the meteorological phenomena which depend on the turbulence of the lower atmosphere, to show how they depend on one another, and to demonstrate some numerical relationships which exist between them.

The transference of heat, water vapour and momentum by means of eddies has been discussed by the present writer in a previous paper.* It was shown there that the effect of turbulent motion on the atmosphere is to endow it with a power of transmitting heat in much the same way as a solid possessing a large coefficient of conductivity.

The temperature, however, which enters into the equations of conductivity in the atmosphere is potential temperature,† instead of being the actual temperature, as it is in the case of the equations for flow of heat in a conducting solid. Thus, if the turbulent air is at a uniform temperature, heat will be transmitted downwards because, under those conditions, the potential temperature increases with height.

The power possessed by the atmosphere in virtue of its turbulence of transmitting heat and momentum may be represented by the symbol K, where K is proportional to the velocity and to the scale of the turbulence. As a rule the atmosphere is stratified in the sense that temperature and velocity vary much more rapidly in a vertical than in any horizontal direction. In this case the rate at which heat flows into unit volume of air is $K\rho\sigma\, \delta^2\theta/\delta z^2$, where θ is the potential temperature, ρ the density and σ the specific heat of air, and z is height measured from the ground. The rate at which momentum parallel to the horizontal axis x is communicated to unit volume is $K\rho\, \delta^2 u/\delta z^2$, where u is the mean component of velocity parallel to the axis x, and the quantity K should, according to this theory, be the same in the two cases. Roughly K may be taken as equal to $\frac{1}{2}wd$, where w represents the mean vertical component of velocity due to the turbulence, and d represents roughly the mean vertical distance through which any portion of the atmosphere is raised or lowered while it forms part of an eddy till the time when it breaks off from it, and mixes with the surroundings. This may be taken to be roughly equal to the diameter of a circular eddy.

* "Eddy Motion in the Atmosphere," 'Phil. Trans.,' A, vol. 215, p. 1, 1915.

† The potential temperature of the air at any height is the temperature to which it would be reduced by expanding or compressing it adiabatically to a standard pressure.

In the paper already referred to, values of K have been found for the turbulence of air blown over the sea, from the measurements of the temperature over the Great Banks of Newfoundland. A few simultaneous measurements of wind velocity over the same area tend to show that the value of this K involved in the equations which represent the transference of momentum is of the same order of magnitude as the K involved in the heat transference equations. The average value obtained for K by these measurements was 3×10^3 in C.G.S. units.

On the other hand, observations taken by means of pilot balloons over Salisbury Plain, of the K involved in the momentum transference equations indicate that the value of K on land is much greater. The mean value was of the order 5×10^4. In 1914 I did not know of the existence of any observations which would enable me to calculate the value of the K involved in the equations for heat transference through the atmosphere over the land, but I have since found, in the 'Annales du Bureau Central Météorologique de France,' for 1894, a series of temperature measurements made at various heights on the Eiffel Tower, which supply the data necessary for calculating this quantity. It will be seen later that these measurements indicate that the value of K over the land is of the order 10×10^4, but that it varies considerably with the time of year, and also, to a certain extent, with the height above the ground.

Temperature Observations on the Eiffel Tower.

During the five years 1890–94 hourly observations of temperature were taken at three different heights on the Eiffel Tower, at 123, 197, and 302 metres above ground. The results have been given by M. Angot, who discussed the observations, in the form of the mean temperature of the air for each hour in the day for each month in the year. Each figure given by M. Angot is, therefore, the mean of about 150 observations.

On examining the rise and fall in temperature during the day at the various stations on the Eiffel Tower, it will be found that the temperature may be represented very approximately by a simple harmonic function of the time. That is to say, the curve which represents the variation in temperature during the day is approximately a sine curve. A specimen curve (fig. 1) has been picked out at random from the observations, and the true sine curve which most nearly represents the real curve has been drawn in as a dotted line beside it. It will be seen that no serious error will result from taking the sine curve instead of the true curve for the purposes of calculation, and it will be found that this introduces considerable simplification into the calculations.

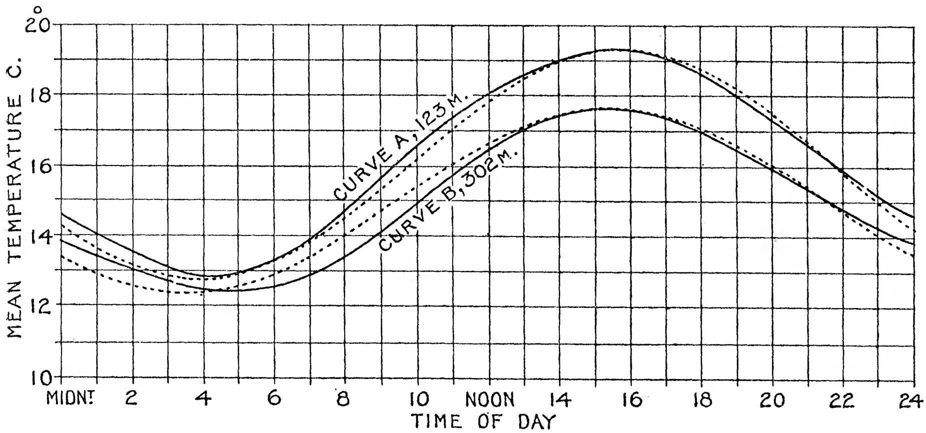


FIG. 1.

Table I.—Amplitude in Centigrade Degrees of the Daily Variation of Temperature at Various Heights.

Month.	Terrace of the Bureau Météorologique.	Eiffel Tower.		
	18 metres.	123 metres.	197 metres.	302 metres.
	°	°	°	°
January	2·99	2·46	1·93	1·31
February	4·52	3·92	3·22	2·30
March	6·46	5·67	4·90	3·90
April	8·45	7·15	6·39	5·12
May	7·76	6·40	5·63	4·82
June	7·76	6·48	5·86	5·21
July	7·57	6·22	5·59	4·98
August	8·06	6·94	5·94	5·12
September	7·95	6·48	5·47	4·34
October	5·61	4·53	3·66	2·78
November	3·24	2·79	2·32	1·55
December	2·74	2·33	2·04	1·40

In Table I are shown the amplitudes of the daily variation in temperature for the various months of the year at various heights above the ground. As might be expected, the daily range in temperature decreases with the height above the ground, and the rate at which this range decreases evidently depends on the amount of turbulence in the atmosphere. These observations of daily range will now be used to calculate the value of K for each month in the year. It will be found that the calculations are very greatly simplified if it is assumed that K is constant at all heights. This assumption will therefore be made in the first place, and afterwards the effect of varying K will be considered.

If z represents the height above the ground, the equation for convection of heat by means of turbulence is $\frac{\delta}{\delta z}(K\rho\sigma\,\delta\theta/\delta z) = \rho\sigma\,\delta\theta/\delta t$, where $\delta\theta/\delta t$ represents the rate of increase in temperature with time.

If K be independent of z this becomes

$$K\,\partial^2\theta/\partial z^2 = \partial\theta/\partial t. \qquad (1)$$

A solution of (1) is

$$\theta = Ae^{-bz}(2\pi t/T - bz), \qquad (2)$$

where

$$b^2 = \pi/TK. \qquad (3)$$

If the daily variation of temperature at the height $z = 0$ is represented by $\theta = A\sin(2\pi t/T)$, where $T = 24$ hours or 86,400 sec., the daily range is 2A.

From (2) it will be seen that the ratio of the daily ranges, R_1 and R_2, at two heights z_1 and z_2, is

$$R_1/R_2 = (2Ae^{-bz_1})/(2Ae^{-bz_2}) = e^{-b(z_1-z_2)}.$$

Hence $$b(z_2-z_1) = \log_e R_1 - \log_e R_2.$$

Hence, if the ratio of the daily ranges at two heights which differ by an amount h is known, the quantity b may be found from the equation

$$b = (\log_e R_1 - \log_e R_2)/h. \qquad (4)$$

The value of K may then be found from equation (3).

In Table I are given the observed daily ranges at three stations on the Eiffel Tower at heights of 123, 197, and 302 metres above the ground, and also the daily range at a station on the terrace of the Bureau Météorologique at a height of 18 metres above the ground. If K were independent of z, it could be found by taking the ratio of the daily ranges at any two heights, and the same result would be obtained whichever pair of ranges were taken. If, therefore, all the values of K obtained by applying equations (3) and (4) to various pairs of stations at different heights are the same, we are justified in assuming that this is the true value of K.* On the other hand, if it be found that the values of K found in this way diminish as the mean height of the air between the two stations increases, it seems justifiable to assume that K diminishes with height, while, if the values of K obtained in this way are found to increase, the real value of K increases.

This method of calculation will not give the true values of K, unless K is found to be constant or nearly constant at all heights, but it may be taken as giving, qualitatively, certain general results concerning the changes in the

* Strictly speaking, this is only true if the stations extend up to such a height that the daily variation in temperature is small compared with the daily variation near the ground, but no considerable error is likely to arise owing to the fact that this condition is not fulfilled.

amount of turbulence at various heights above the ground during the course of the year. In Table II are given the results of applying equations (3) and (4) to the daily ranges in temperature given in Table I, in order to find the mean value of K in different months of the year.

Table II.—Mean Values of K between Various Heights from 18 to 302 Metres above the Ground.

Month.	1. 18 to 302 metres.	2. 123 to 302 metres.	3. 197 to 302 metres.	4. 18 to 123 metres.
January	$4 \cdot 3 \times 10^4$	$2 \cdot 9 \times 10^4$	$2 \cdot 7 \times 10^4$	11×10^4
February	$6 \cdot 4 \times 10^4$	$4 \cdot 1 \times 10^4$	$1 \cdot 6 \times 10^4$	20×10^4
March	$10 \cdot 5 \times 10^4$	$8 \cdot 3 \times 10^4$	$7 \cdot 7 \times 10^4$	24×10^4
April	$10 \cdot 2 \times 10^4$	$10 \cdot 5 \times 10^4$	$8 \cdot 2 \times 10^4$	14×10^4
May	$12 \cdot 9 \times 10^4$	$14 \cdot 4 \times 10^4$	$16 \cdot 7 \times 10^4$	11×10^4
June	$18 \cdot 3 \times 10^4$	$24 \cdot 4 \times 10^4$	$28 \cdot 8 \times 10^4$	12×10^4
July	$16 \cdot 7 \times 10^4$	$23 \cdot 4 \times 10^4$	$30 \cdot 1 \times 10^4$	13×10^4
August	$14 \cdot 6 \times 10^4$	$13 \cdot 1 \times 10^4$	$19 \cdot 6 \times 10^4$	18×10^4
September	$8 \cdot 0 \times 10^4$	$7 \cdot 2 \times 10^4$	$7 \cdot 5 \times 10^4$	10×10^4
October	$5 \cdot 9 \times 10^4$	$4 \cdot 9 \times 10^4$	$5 \cdot 3 \times 10^4$	9×10^4
November	$5 \cdot 4 \times 10^4$	$3 \cdot 2 \times 10^4$	$2 \cdot 5 \times 10^4$	18×10^4
December	$6 \cdot 5 \times 10^4$	$4 \cdot 4 \times 10^4$	$2 \cdot 8 \times 10^4$	15×10^4
Mean	$10 \cdot 0 \times 10^4$			

The most noticeable feature of the Table is the way in which the turbulence appears to decrease with height in the winter and to increase in the summer. In June and July, for instance, the mean values of K in the whole height of the tower, from 18 to 302 metres, are 18 and 17×10^4, while the mean values from 197 to 302 metres are 29 and 30×10^4 respectively. In the winter months—November, December, January, and February—the mean values of K from 18 to 302 metres are 5·4, 6·5, 4·3, and $6{\cdot}4 \times 10^4$, while the mean values from 197 to 302 metres are 2·5, 2·8, 2·7, and $1{\cdot}6 \times 10^4$ respectively.

The explanation of this diminution of turbulence at the top of the Eiffel Tower in the winter must be looked for in the temperature gradient. The mean temperature gradient up to 300 metres is considerably less than the adiabatic gradient in the winter, and the number of occasions when the adiabatic gradient is reached is comparatively small. A gradient less than the adiabatic has a tendency to prevent the spontaneous formation of turbulence, and to suppress it when formed by any outside agencies, such as obstacles on the ground.

In the summer the mean temperature gradient in the first 300 metres is much more nearly adiabatic, and the number of occasions when it reaches

the adiabatic gradient is large. A gradient equal to the adiabatic gradient has a tendency to encourage the spontaneous formation of turbulence. Under these circumstances, an increase in the value of K with height is to be expected, because K is roughly proportional to the vertical component of turbulent velocity and to the diameters of the eddies. It is to be expected that the eddies will increase in size as the height above the ground increases, because they have more room to grow. There will, therefore, be a tendency for K to increase with height.

Looking at the first and third columns in Table II, it will be seen that the mean values of K between 18 and 302 metres and between 197 and 302 metres both have a maximum in the summer and a minimum in winter, but the variation is much greater in the latter case than in the former. It is a matter of some interest, therefore, to find out whether the value of K near the ground shows any marked monthly variation. In column 4, Table II, is given the result of applying equations (3) and (4) to the daily ranges of temperature at 18 and 123 metres, but it must be remembered that the errors due to the method used in deducing K are greater near the ground, where the most rapid variations are likely to occur, than they are higher up. Accuracy in the figures of column 4 is therefore not to be expected.

It will be seen that the values of K near the ground appear to vary in a haphazard manner, but that they show no marked monthly variation of the type exhibited by the figures in all the other columns. It seems, therefore, that the effect of the mean temperature gradient, which reduces the turbulence at 300 metres in the winter and allows it to increase in the summer, does not have any marked effect on the mean amount of turbulence near the ground. This must be governed almost entirely by wind velocity, which shows no marked monthly variation, and by the nature of the ground.

On the other hand, it will be seen later that the daily variation in temperature gradient near the ground has a very marked effect on the turbulence near the ground.

Identity of K *found from Temperature Measurements with* K *found from Wind Measurements.*

On looking at column 1, Table II, it will be seen that the mean value of K for the turbulence over Paris, as calculated from the Eiffel Tower temperature measurements, is about 10×10^4. It was pointed out on p. 138 that the mean value of K for the turbulence over Salisbury Plain, as calculated from wind-velocity measurements, is about 5×10^4. It is to be

expected, from the nature of the ground, that the turbulence over Paris would be greater than the turbulence over Salisbury Plain.

Quite recently, since finishing the work described above, I have seen a paper by Dr. F. Åkerblom,* in which he uses the change in wind direction between the top and the bottom of the Eiffel Tower to find the coefficient of viscosity of the atmosphere due to turbulence. This quantity, which is, of course, equal to K/ρ, was found to be about 85 C.G.S. units in the winter and 115 in the summer. Åkerblom finds, therefore, that the mean value of K in the height of the Eiffel Tower is greater in the summer than in the winter, but the difference is considerably less than that indicated in the first column of Table II. The mean value obtained for K/ρ is given by Dr. Åkerblom as 95 C.G.S. units. Taking ρ as 0·00125, the value of K over Paris, calculated from wind-velocity measurements, is therefore 95/0·00125, or $7{\cdot}6 \times 10^4$ C.G.S. units.

The agreement between this and the value of 10×10^4 obtained above from temperature measurement is quite as good as could possibly be expected, when it is remembered that neither Åkerblom's nor the present writer's equations are rigidly applicable under the conditions which actually occur in the atmosphere. It is sufficiently good, at any rate, to provide a satisfactory confirmation of the theory that momentum and heat are transmitted by the same agency, and that the behaviour of the lower atmosphere in regard to heat transmission can be calculated from observations of the retardation of the lower layers of the earth's atmosphere by the friction of the ground.

Daily Variation in Wind Velocity.

We have seen how a study of the change in wind velocity with height and of the diurnal variation in temperature leads to a knowledge of the amount of turbulence in the atmosphere near the ground. We have seen, also, how much the turbulence is reduced in the winter by the smaller average temperature gradient which is characteristic of that part of the year. So far, however, no mention has been made of what is by far the most noticeable feature of the meteorological effects of the turbulence of the lower atmosphere, namely, the daily variation of wind velocity. The remainder of the paper will be devoted to discussing this variation. It will be shown that the knowledge which had been gained concerning turbulence is sufficient to explain all the facts which recent observations have brought to light concerning the daily variation in wind velocity at various heights above the ground and at various seasons of the year.

It is well known that the wind near the ground is usually less strong at

* F. Åkerblom, 'Nov. Act. Soc. Scient.,' Upsala, 1908.

night than it is in the day time. Observations taken on mountains and on high buildings show that at comparatively small heights above the earth this variation in wind velocity is much less than it is quite close to the ground. The Eiffel Tower observations show that at a height of 1000 feet the daily variation in wind velocity is reversed, so that it has a maximum at night and a minimum during the day.

Quite recently the diurnal variation in wind velocity has been made the subject of an elaborate series of observations by Dr. Hellmann.* A brief description of his results will form the best introduction to the more theoretical discussion which will follow later.

In a piece of flat meadow-land Dr. Hellmann set up three anemometers at heights of 2, 16, and 32 metres above the ground, and the results of one year's observations are dealt with in his paper.

During periods when strong winds were blowing, all three anemometers showed a maximum velocity in the middle of the day and a minimum during the night. The daily variation in light winds was of a different character. At 2 metres there was a maximum in the day and a minimum at night. At 16 and 32 metres, however, there were maxima in the middle of the day, and also in the middle of the night, with minima during the morning and afternoon. At 16 metres these two maxima were about equal, while at 32 metres the night maximum was greater than the day maximum.

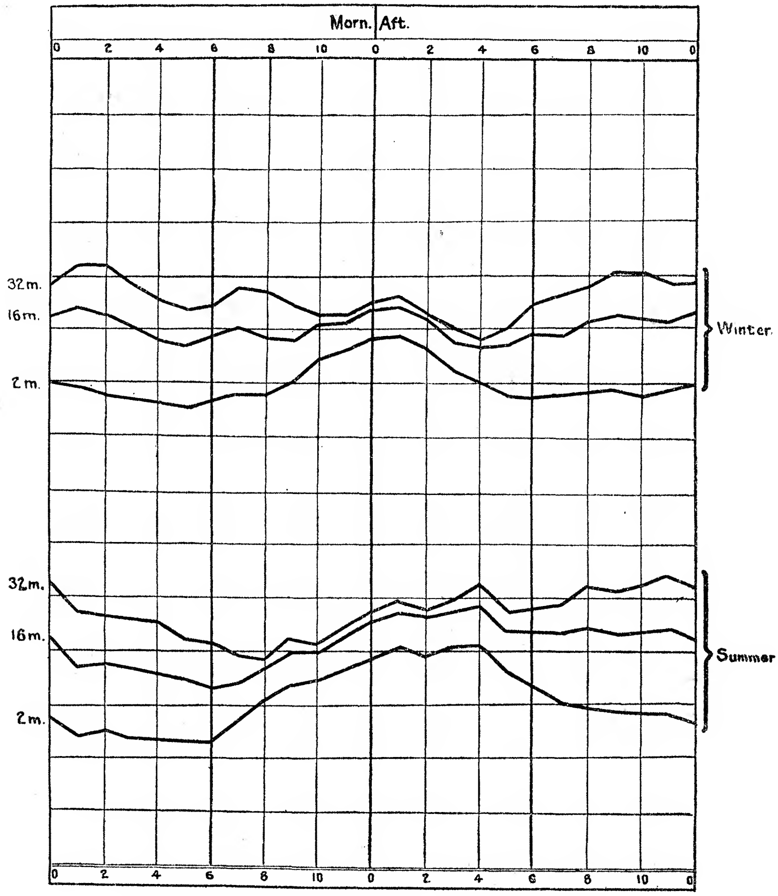
On examining the seasonal effect, it was found that there was a distinct tendency in summer for the day maximum at 16 metres to be greater than the night maximum, while in the winter the two maxima were about equal.

These results are shown graphically by means of curves in figs. 2 and 3. It will be seen that in light winds the height at which the day and night maxima are equal is about 16 metres in winter and between 16 and 32 metres in summer. At greater heights the wind is greatest in the middle of the night, while at smaller heights it is greatest in the middle of the day. This change in the type of the daily variation in wind velocity will be referred to as a "reversal."

Dr. Hellmann's results are confirmed by his analysis of the daily variation in wind at the observatory at Potsdam, where the anemometer is placed at a height of 41 metres above the ground. In fig. 4 is shown the variation in wind velocity for light and for strong winds in the summer and in the winter. It will be seen that for strong winds the maximum occurs in the middle of the day, at all times of the year. In the case of light winds there

* "Uber die Bewegung der Luft in den untersten Schichten der Atmosphäre," 'Meteorologische Zeitschrift,' January, 1915.

is a minimum in the middle of the day in the winter, while in the summer there is a small maximum in the middle of the day, and a much larger maximum in the middle of the night. It appears, therefore, that the height at which the reversal in type of daily variation occurs is greater than 41 metres for strong winds, but that for light winds it is less than



Daily Variation of Wind Velocity with Light Winds.

Fig. 2.

41 metres. During light winds in winter the height of the reversal is so much less than 41 metres that the day maximum has altogether disappeared at that height. In the summer, however, the height of the reversal is evidently much nearer to 41 metres, for the day maximum is still quite well developed at that height. These results agree well with Dr. Hellmann's observations.

To account for the facts brought to light by these observations, Dr. Hellmann

mentions the theory of Espy and Köppen, according to which both types of daily variation are the results of ascending and descending currents produced by the heating of the earth in the day time. The descending currents carry the upper wind down to the ground, thus increasing the surface wind, while the ascending currents carry the more stagnant air at

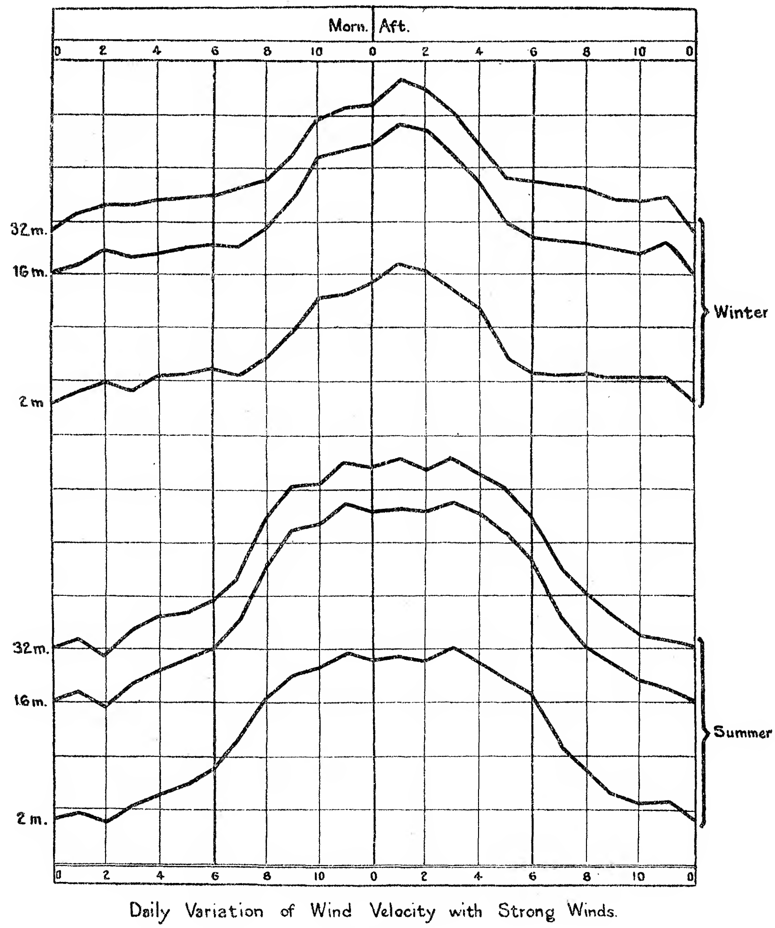


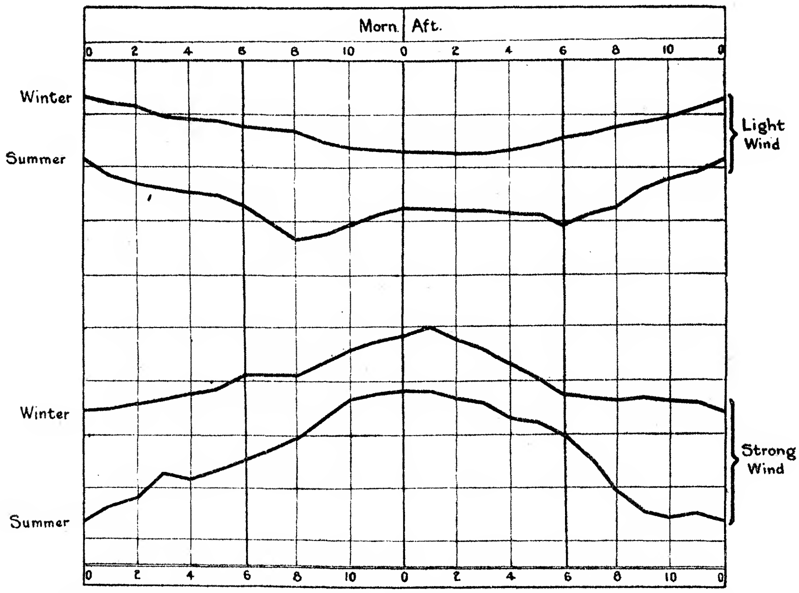
Fig. 3.

the surface up into the higher layers, an action which reduces the velocity in those regions.

This theory appears to contain some elements of truth, but in several ways it is very unsatisfactory. In the first place it is purely qualitative. Presumably, the height at which the reversal takes place should, according to this theory, be some definite fraction of the height to which the vertical currents produced by the heating of the ground extend. In order, therefore,

that the theory may account for Dr. Hellmann's observations, it is necessary to assume that the vertical currents, due to heating of the ground, extend to a much greater height in strong winds than they do in light winds. This is contrary to the commonly accepted idea of vertical heat currents, and is almost certainly untrue.

On examining the Espy-Köppen theory closely, it will be found that it not only fails to give quantitative expression to the phenomena of daily variation in wind velocity, but that it involves implicitly several special assumptions regarding the nature of the vertical currents which convey horizontal momentum from the upper layers to the lower layers of the atmosphere, and



Daily Variation of Wind Velocity in Potsdam with Light and Strong Winds.

FIG. 4.

from these to the ground. Moreover, there is no other evidence that these special assumptions are true. We shall now enquire into the nature of these special assumptions.

The vertical currents in the atmosphere promote an interchange of momentum between the different layers. The action of an increase in the amount of vertical currents in the lower atmosphere is to increase the amount of rapidly moving air brought down from the upper layers to the layers close to the ground, but at the same time it increases the amount of slowly moving or stationary air brought up from the surface of the ground. The velocity of the lowest stratum of air is determined by the balancing of

these two effects, and in order that an increase in the velocity of the wind close to the ground may result from an increase in the amount of the vertical currents, it is necessary to assume that the structure of the vertical currents is such that the increase in velocity due to the extra influx of momentum from above is greater than the decrease due to the extra outflow of momentum into the ground. It has been shown by the present writer in a previous paper* that, with a given amount of eddy motion and a given pressure gradient, there is a certain distribution of wind velocity near the ground, which exists as a steady state when the system has been established long enough for the effect of the initial conditions to have died away. In the Espy-Köppen theory no account is taken of the steady state. In the present paper, on the other hand, it will be assumed that the distribution of wind velocity in the lower atmosphere at any time is the distribution which would result from the gradient velocity and turbulence which exist at that time if the steady state had been reached. This is equivalent to assuming that the lag of the variation in wind velocity behind the variation in turbulence which gives rise to it is small. That this is sufficiently near the truth for our purpose is shown by the fact that the maxima in wind velocity occur about the middle of the day, at a time when the turbulence might be expected to be at its maximum.

It will be shown that a daily variation in turbulence, by an amount which might have been expected from our previous knowledge of the subject, is sufficient to explain qualitatively—and also, in a rough way, quantitatively—all the known characters of the daily variation in wind velocity.

We have seen that the effect of turbulence on the distribution of velocity in the lower atmosphere depends on a certain quantity K, which can be regarded as a number which expresses the amount of turbulence. It has been shown also that the mean distribution of wind velocity in the lower layer of the atmosphere is very nearly the same as that in an ideal atmosphere in which K is constant at all heights. We shall therefore find how the wind velocity at various heights would vary in the ideal case when its pressure gradient remains constant and K is constant at all heights but has a daily variation in magnitude.

For this purpose it is necessary to take account of the fact that the frictional force of the wind over the ground is proportional to the square of the wind velocity.† The magnitude of the frictional force has been calculated for Salisbury Plain. It was shown that over the grassy land of which Salisbury Plain is composed, the frictional force, F, is 0·0023 ρQ_s^2,

* "Eddy Motion in the Atmosphere," 'Phil. Trans.,' A, vol. 215 (1915).

† 'Roy. Soc. Proc.,' A, vol. 92, p. 198 (1916).

where Q_s is the wind velocity close to the ground, and ρ is the density of the air. Using this value for F it may be shown that

$$\frac{F}{\rho} = 0{\cdot}0023\, Q_s^2 = 2\,KQ_G \sin\alpha/B, \tag{5}$$

where α is the angle between the wind at the ground level and the gradient direction, Q_G is the gradient velocity, K is the coefficient of eddy diffusion already referred to, and B is equal to $\sqrt{(\omega \sin\lambda/K)}$,* where ω is the angular velocity of rotation of the earth, which is numerically equal to 0·000073, and λ is the latitude of the place in question, which in the case of Salisbury Plain is 50° N., so that $\sin\lambda = 0{\cdot}77$. Equation (5) may be transformed into the form

$$\frac{1}{BQ_G} = \frac{1}{2\sin\alpha}\left(\frac{Q_s}{Q_G}\right)^2\left(\frac{0{\cdot}0023}{0{\cdot}000073 \times 0{\cdot}77}\right), \qquad \text{but}\dagger\ Q_s/Q_G = \cos\alpha - \sin\alpha.$$

Hence
$$\frac{1}{BQ_G} = \frac{20{\cdot}4}{\sin\alpha}(\cos\alpha - \sin\alpha)^2.$$

The values of $1/BQ_G$ may therefore be tabulated for a series of values of α. These are given in Table III.

Since B depends only on K, it might be simpler to consider the variations in wind velocity at various heights when B rather than K undergoes a daily variation; but since the numerical value of K has been measured in several cases already, it is convenient to have a Table showing the relationship between K and α. The quantity K/Q_G^2 is a function of α, and its values are given in the last column of Table III.

Table III may now be used in conjunction with the equations given on pp. 15 and 16 of "Eddy Motion in the Atmosphere," to find the wind velocity at any height for a given gradient velocity and a given value of K. The numerical work is laborious, but the results can be shown simply by means of a series of curves. These curves are shown in fig. 5. The abscissæ represent the wind velocity as fractions of the gradient, while the ordinates represent the quantity z/Q_G, where z represents the height above the ground. If the curves be regarded as giving the variation in wind velocity with height, the scale of the ordinates will depend on the gradient velocity. Two scales of height in metres have been drawn on the right-hand side of the figure, corresponding with the two gradient velocities of 4·6 and 15·6 metres per second, which correspond with the classes "light" and "strong" winds for which mean values of K have been calculated.‡

* "Eddy Motion in the Atmosphere," *loc. cit.*, p. 15.

† *Ibid.*, p. 16.

‡ 'Roy. Soc. Proc.,' A, vol. 92, p. 198 (1916).

Each of the curves in fig. 5 represents the distribution of wind velocity at various heights with a given value of K and Q_G. When K undergoes a daily variation, the daily variation in wind velocity is given by the variation in the abscissæ of the different curves for a fixed ordinate.

Table III.

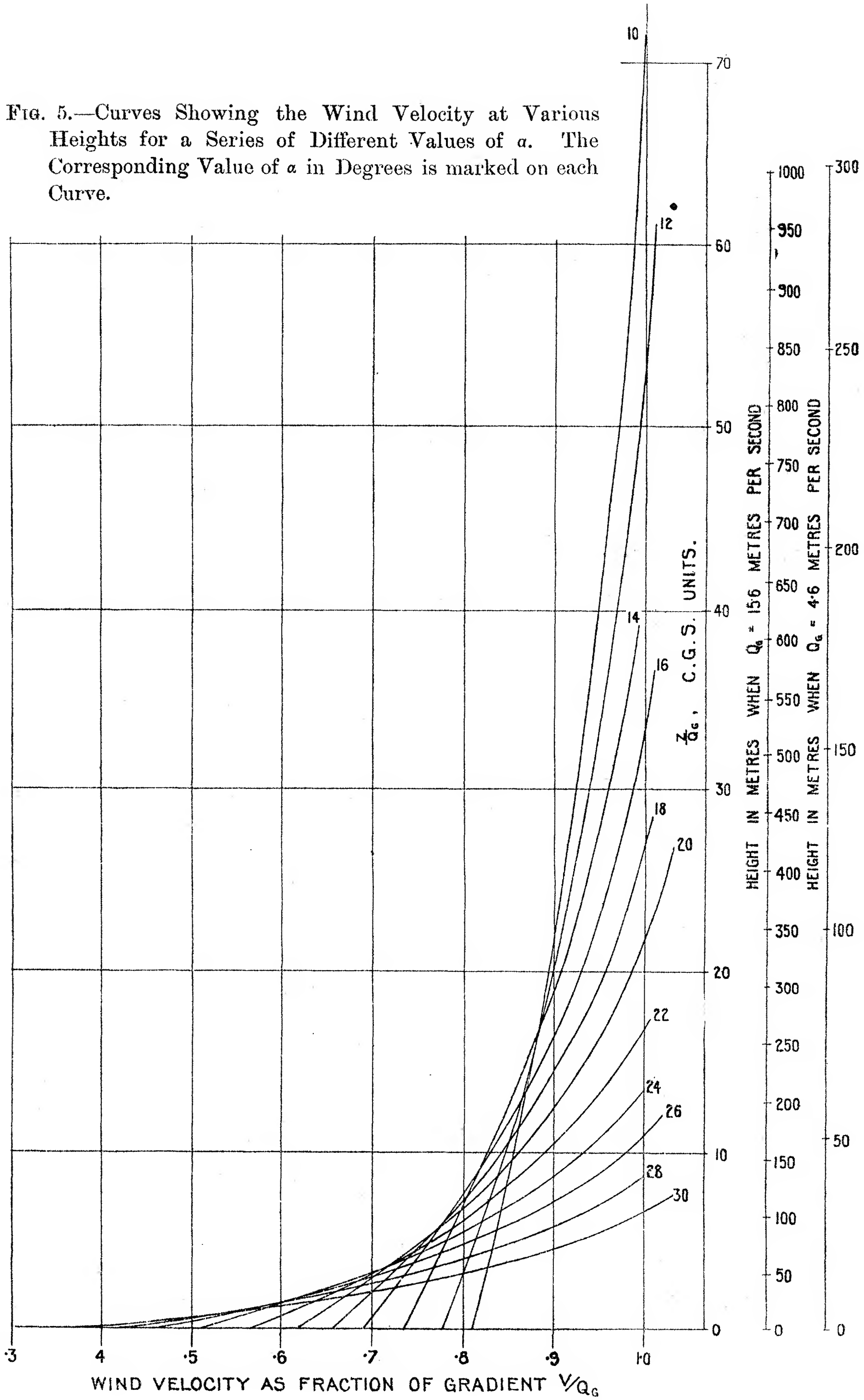
α.	$1/DQ_G$.	K/Q_G^2.
Degrees.	C.G.S. units.	C.G.S. units.
4	252	3·54
6	155	1·35
8	106	0·635
10	77·5	0·338
12	58·5	0·192
14	44·8	0·116
16	34·9	0·069
18	27·3	0·042
20	21·9	0·027
22	16·7	0·0156
24	12·9	0·0094
26	9·9	0·0055
28	7·4	0·0031
30	5·5	0·0017
32	3·7	0·00085
34	2·6	0·00038
36	1·7	0·00016

In order to simplify the application of the curves in fig. 5 another set of curves, shown in fig. 6, has been constructed from them. These represent the variation in wind velocity at a given height when α varies owing to the variations in K. Each curve represents the variation in value of V/Q_G for a given value of z/Q_G when α varies from 6° to 30°.

An inspection of the curves of fig. 6 shows that the daily variation in K will account for the character of the daily variation in wind at different heights. Suppose, for instance, that the value of K varies in such a way that $\alpha = 10°$ at midday and 30° at midnight. At heights above that at which $z/Q_G = 15$ the wind will be at a maximum at midnight and at a minimum at midday. At heights for which z/Q_G is equal to or less than 1 there is a maximum at midday and a minimum at midnight. At all intermediate heights there are two maxima, one at midday and the other at midnight. When $z/Q_G = 4$ the maxima at midday and at midnight are about equal for the particular range of K we have chosen.

The several characteristics of the daily variation in wind velocity which this theory indicates are shown in fig. 7. In this figure the value of K has been assumed to vary in a continuous manner so that it is a maximum at midday and a minimum at midnight. Each curve represents the variation

FIG. 5.—Curves Showing the Wind Velocity at Various Heights for a Series of Different Values of α. The Corresponding Value of α in Degrees is marked on each Curve.



in wind velocity at a given height, represented, on an arbitrary scale, by the figure attached to the curve.

It will be seen that this agrees remarkably well with the variation which was actually observed by Dr. Hellmann. In the case of light winds in winter, for instance, the wind velocity at 2 metres was a maximum at midday and remained practically constant from 6 P.M. to 6 A.M. The wind at 16 metres had equal maxima at 1 A.M. and 1 P.M. The wind at 32 metres had a maximum at about 1 A.M. and a smaller maximum about 1 P.M. It also had a

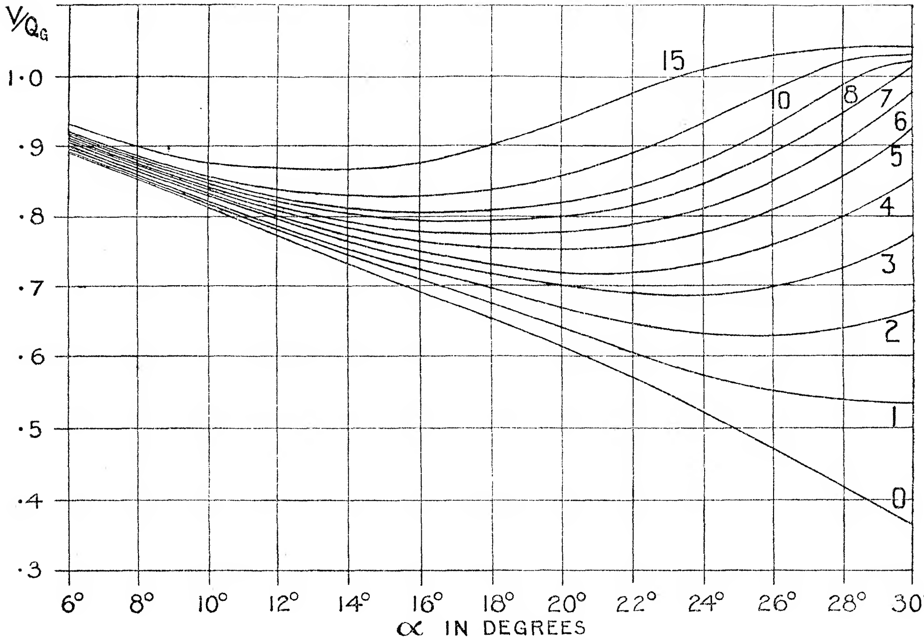


FIG. 6.—Curves Showing the Variation in Wind Velocity at Various Heights above the Ground when the Gradient Velocity remains Fixed and α Varies owing to Variations in K. Each Curve Represents the Variations at a Fixed Height, and the value of z/Q_G which corresponds with that is marked on the Curve in Question.

maximum at about 7 A.M., but this appears to be an irregularity due to the fact that Dr. Hellmann's observations only extend over one year. The curves showing the daily variation of wind velocity at Potsdam, which are based on five years' observations, certainly do not possess the corresponding maximum.

It is interesting to see how far the information which we have obtained in various ways concerning the value of K may be used to set numerical limits to the height to which the ground type of daily variation in wind velocity is likely to extend.

It has been shown* that the mean value of K in the summer in the height

* See Table II.

of the Eiffel Tower is about 15×10^4, while in the winter it is about 5×10^4. These include strong as well as light winds, and night as well as day. Since the value of K must be very low on all clear nights it seems probable that one might expect the mean value of K at midday to be at least as great as 30×10^4. It will be higher for strong winds and lower for light winds, and in the absence of any further data we might guess the values 40×10^4 for strong winds and 20×10^4 for light winds. For the winter the values of K may be taken as one-third of the summer values, because the mean value of K in

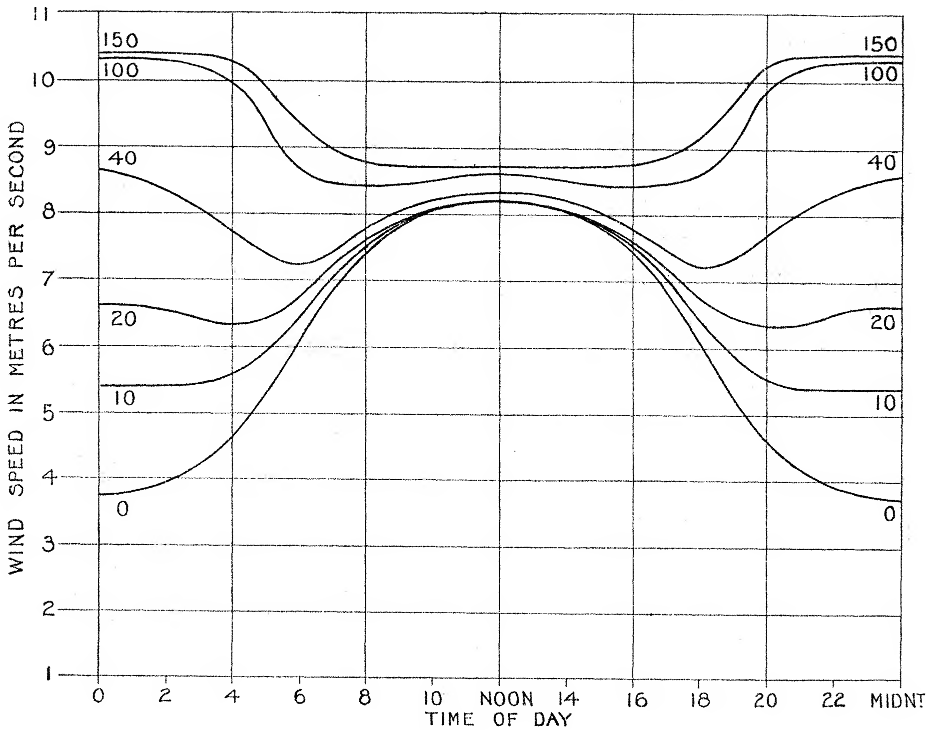


FIG. 7.

winter is one-third of the mean value in summer. Hence they may be taken 13×10^4 and 7×10^4.

The value of K at night is governed by the fact that the temperature gradient is less than the adiabatic gradient, in fact it is frequently reversed near the ground. Under these conditions the stability of the air tends to prevent the formation of turbulence. So far the value of K has not been determined on land under circumstances in which the temperature gradient is known. At sea, however, the value of K, under circumstances when there was an inversion of temperature, was found to be of the order 3×10^3 for moderate winds and 10^3 for light winds. In strong winds at sea, therefore,

we might expect K to be of the order 6×10^3 when there is an inversion of temperature gradient. On land there is likely to be a still greater difference between the values of K for light and for strong winds at night because the air temperature at the ground does not go down so much when the wind is strong as when it is light, whereas the temperature of the air near the sea is practically the same as that of the sea itself, whatever the wind velocity may be.

Taking the values of K given above and taking the mean gradient velocities in light and strong winds as 4·6 and 13·6 metres per second respectively, it will be found from Table III that the corresponding values of α are those set out in Table IV.

From the curves in fig. 6, the value of z/Q_G at which the maxima at midday and midnight are equal may be found. In the case of strong winds in summer, for instance, the variation of α from 12° to 28° gives equal maxima of wind velocity, equal to 0·83 Q_G when $z/Q_G = 4\frac{1}{4}$. This corresponds with a height of 60 metres. At heights lower than this the greatest wind will be at midday, while for greater heights the maximum will be at midnight.

Table IV.

		α at midday.	α at midnight.	z/Q_G.	Height at which maxima at midday and midnight are equal.
		°	°		Metres.
Strong winds	Summer ...	12	28	4	60
	Winter	17		3	50
Light winds	Summer ...	7	26	7	30
	Winter	10		$5\frac{1}{2}$	25

The heights at which the reversal in the character of the daily range occurs are given in the last column. It appears, therefore, that for light winds the reversal might be expected at heights between 25 and 30 metres. In strong winds, however, the type of daily variation in which the wind velocity is a maximum at midday would extend up to 50 or 60 metres. If, as was anticipated above, the value of K at midnight is considerably greater in strong winds than we have taken it to be, then the height of the reversal of the type of daily range would be much higher than 50 or 60 metres.

On referring to the curves of figs. 2, 3, and 4, it will be seen that these theoretical conclusions agree with Dr. Hellmann's observations. He found the reversal to occur in light winds at about 16 metres in the winter and

about 32 metres in the summer. In the case of strong winds there was no reversal in the first 32 metres.

In the case of Potsdam observations, it was found that the height of the reversal was less than 41 metres for light winds and greater for strong winds. In the case of light winds in summer, there is a maximum at midday at Potsdam, but it is smaller than the maximum which occurs at midnight.

It appears, as a result of these calculations, that a variation in K by an amount which fits in well with all the other known data concerning the turbulent motion of the air near the ground is sufficient to explain, both qualitatively and quantitatively, all the facts concerning the daily variation of wind velocity at different heights above the ground which are brought to light by Dr. Hellmann's observations.

The Discharge of Gases under High Pressures.

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Introduction.

This work was undertaken, following up a suggestion made by Lord Rayleigh.* De St. Venant and Wantzel in 1839 stated, as a result of their experiments, that when gas was discharged through an orifice from a vessel in which the pressure was p_0, into one in which the pressure was p_1, then the rate of discharge was sensibly constant from $p_1 = 0$ upwards to $p_1 = 0{\cdot}3p_0$ or $0{\cdot}4p_0$, but then, as p_1 further increases, the discharge falls off, slowly at first, afterwards with great rapidity.

In 1885 Osborne Reynolds quoted some experiments which seemed to show that the flow remained constant from $p_1 = 0{\cdot}50p_0$ or $0{\cdot}53p_0$. He explained this by pointing out that the maximum "reduced velocity" occurs when the actual velocity coincides with that of sound under the conditions then prevailing, as then the effect of a further reduction of pressure in the recipient vessel cannot be propagated backwards against the stream. If $\gamma = 1{\cdot}408$ this argument suggests that for a nozzle ending abruptly at the narrowest part, the discharge reaches a maximum when the pressure in the

* 'Phil. Mag.,' vol. 32, p. 178 (1916.)